

Editorial on “TP-based Systems and Education”

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This special issue collects contributions to the working group “Theorem-Prover based Systems for Education (eduTPS)”¹ at CADGME’12², the conference on “Computer Algebra and Dynamic Geometry Systems in Mathematics Education” at Novi Sad in June 2012.

The collection of six papers seems to be sparse in comparison to the wide scope of *eduTPS*, which spans over three academic disciplines, (1) Mathematics Education, (2) Computer Theorem Proving (TP) and (3) E-Learning. The “tag clouds” in the figure on p.?? below characterise these disciplines as fairly disjoint.

The scope of *eduTPS* and of this special issue, however, is not just the union of the three disciplines; rather, the scope is determined by concrete promises of software based on technology from the discipline of *Computer Theorem Proving*, i.e. TP technology, for education. With respect to this focus the six papers are nicely distributed over the three disciplines.

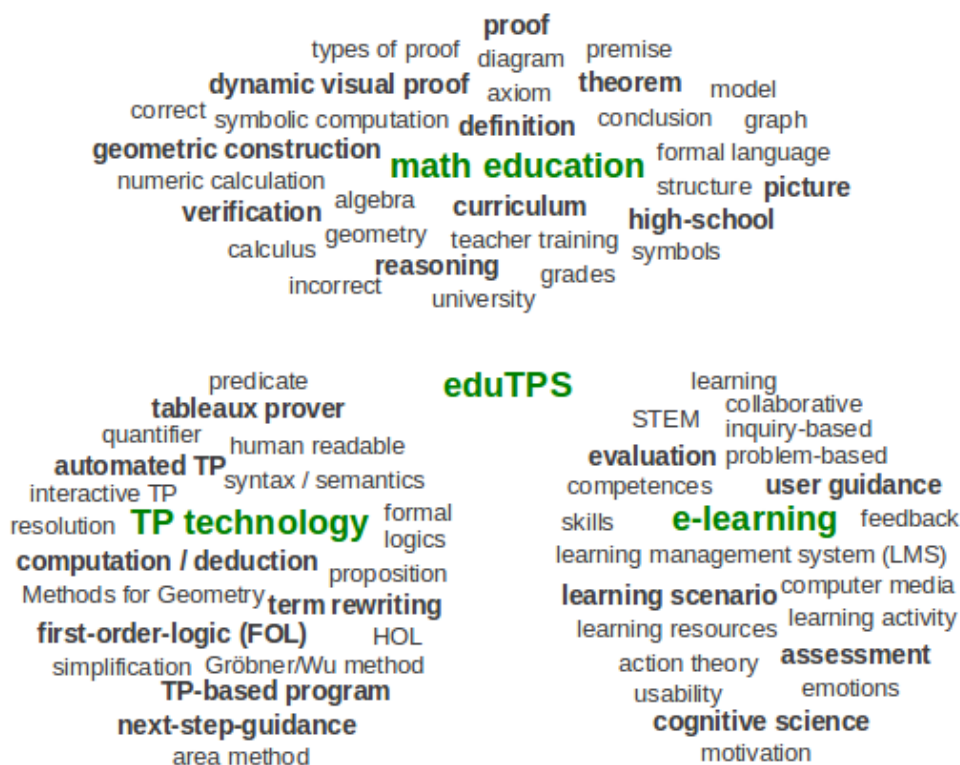
The focus has two immediate consequences for present and future practice of mathematics education and more generally, of education in “Science, Technology, Engineering and Mathematics” (STEM), where mathematics provides the methodological core; one consequence are *TP-services for educational software* and the other consequence is *software support for interactive proof*:

TP-services for educational software are mentioned first in order to face the common objection “Computer Theorem Proving is too hard for high-school” — right, the intention of eduTPS is different: provide TP-based services for

- checking user-input (within a logical *context*, user-input states a lemma)
- guidance to a next step (using the above *context* for “next-step-guidance”)

¹<http://sites.dmi.rs/events/2012/CADGME2012/mformats.html>

²<http://sites.dmi.rs/events/2012/CADGME2012>



- inspection of underlying mathematics knowledge (exploiting the LCF paradigm ³)

... services which hide sophisticated TP technology behind user interfaces of various software, which might include DGS in the future. Such software is expected to be usable for students upwards from the level encountering variables first time (usually at the age of 13 years at high-school).

The paper *“Interactive Course Material by TP-based Programming”* by Jan Ročník focuses on such technology in a case study: The course material is a more or less an automatic consequence of a novel kind of programs, which involve TP technology completely hidden from learners, but challenging for the programmer. The programs provide the TP-based services mentioned above. And these services in turn provide a powerful base for developing adaptive user-guidance as shown in the paper *“Error-Patterns within Next-Step-Guidance in TP-based Educational Systems”* by Gabriella Daróczy.

Not only usability is affected by TP-based technology, also the scope of mathematics is extended to activities which were not possible without mathematical software. Jakub Jareš and Pavel Pech show such activities in their paper *“Exploring loci of points by DGS and CAS in teaching geometry”*. This paper also identifies learning scenarios, which raise motivation for formal proof in a natural way — in the paper still without consistent software support, which is expected to arise from cooperation between the mentioned three disciplines in the near future.

³Following the LCF paradigm, some TPs build up all math knowledge from first principles in a human readable format; see for instance <http://isabelle.in.tum.de/dist/library/HOL>.

Software support for interactive proof is reliably grounded on formal calculus in TP — whereas traditionally proof is *not* taught as a formal calculus in general ⁴. Rather, students adopt the habits of their academic environment and copy the way of proving from their academic teachers. Evidently this results in a lack of confidence for many students and a lack of control of what they are doing ⁵. High-school is affected by these academic traditions such, that proof is considered too comprehensive for most students in spite of curricula explicitly comprising mathematical proof.

The paper “*Dynamic Visual Proofs’ in DGS*” by Irena Štrausová and Roman Hašek reflects this “non-formal” state of proving in mathematics education, while it presents an innovative approach promising to be attractive for many more students at high-school. Their “proofs” (in quotations marks, as seen from a perspective of TP and mathematics) subtly exploit DGS’ interactive features.

Actually, in leading DGS there is automated TP under construction, which promises to connect rigorous formal proof with “dynamic visual proof” — the upcoming years will show, if DGS will succeed handling these different kinds of proofs in parallel, similar to how they already handle lines as geometric objects and as analytic terms in two variables, for instance.

The paper “*Recovering Intuition from Automated Formal Proofs using Tableaux with Superdeduction*” by David Delahaye and Melanie Jacquel already goes a step further: it presents technology which can support *interactive* proof directly — proof not by translation of geometry into an algebraic representation (Gröbner bases etc.), but proof by handling geometric objects in some axiom system of geometry.

Since the technology presented by Delahaye and Jacquel is not restricted to geometry, their work directly supports the raise of “educational TPs”, which overlay the overwhelming power of present TPs with support for “intuition” and probably establish continuous software support from high-school to university.

A survey on TP-based systems for education is *not* provided in any of the six papers; rather, there is a synopsis “*On the Emergence of TP-based Educational Math Assistants*” by Walther Neuper.

A survey on TP-based systems for education is covered by the “Proceedings of the First Workshop on ‘CTP Components for Educational Software’ (THedu)” in Wrocław, Poland, 31th July 2011 ⁶. The *THedu* workshops address experts in TP who are interested in education and e-learning, while here there is a better balance: The *eduTPS* workshops address experts in math education and experts in e-learning interested in TP-technology as well as TP-experts interested in education.

With that scope in mind the papers of this issue are well balanced between the disciplines of (1) math education, (2) TP-technology and (3) e-learning : Each paper addresses a second discipline beyond a primary one: (1) is primarily addressed by Hašek, Jareš, Pech and Štrausová, (2) by Daróczy, Delahaye, Jacquel and Ročník and (3) by Daróczy and Neuper.

E-learning as described in the figure above apparently is addressed least by the papers: Explicit construction of learning scenarios, of evaluation strategies, assessment, etc would require software

⁴Courses in formal logics are usually separated from regular practice of proving; also, this kind of courses is more frequently found in Computer Science than in Mathematics.

⁵For talented students this kind of learning might be most proficient in a supportive academic environment, because it most efficiently fosters ingenuity required for proving.

⁶<http://rvg.web.cse.unsw.edu.au/eptcs/content.cgi?THedu11>

ready for classroom use, which is not yet the case with TP-based systems. However, explicit learning scenarios and evaluation are indispensable in proposals for funding. Respective calls are dominantly refereed by experts in e-learning, while experts in math education and in TP-technology seem under-represented in respective funding agencies.

Summarising this editorial for the Special Issue, the aims of *eduTPS* are reminded as:

1. Provide an forum for mathematics educators to exchange experiences on using (prototypes of) TP-based systems in educational practice.
2. Foster contacts of educators and e-learning experts with TP experts in order to develop concrete ideas for future software development.
3. Give opportunity for TP experts to gain feedback from educators and e-learning experts on their technologies' relevance for education.

The Program Committee of *eduTPS* hopes to contribute herewith to these aims. The Committee is aware of many colleagues from the three targeted domains, who's expertise would be highly welcome for contribution; these colleagues and all other experts interested in the topic are cordially invited for cooperation.

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